V. S. Spirídonov, S. V. Belov, and O. V. Kirikova

A method is proposed for calculating the narrowest cross sections of pore channels averaged over the filtration surface. The calculated results are confirmed by experimental data.

The pores in reticular porous materials are channels of variable cross section. The maximum capillary pressure developed in such channels and the smallest solid particle which can be removed by the channels from gas and liquid flows due to the "sieve" effect are determined by the dimensions of their narrowest cross section $\delta_{i}^{m}$. In any porous material, the dimensions of the cross section $\delta_{i}^{\mathrm{m}}$ of pore channels change within a certain range $\delta_{\min }^{\mathrm{m}} \delta_{\max }^{\mathrm{m}}$, with the probability density $\mathrm{f}=\mathrm{f}\left(\delta^{\mathrm{m}}\right)$. The mean size of these cross sections $\delta_{\mathrm{av}}^{\mathrm{min}}$ Is maxually used as the characteristic dimension of the internal structure of filtering baffles, heat-pipe wicks, etc. The value of $\delta \mathrm{m}_{\mathrm{a}}$ is found experimentally by expelling fluid from pores [1]. Theoretical determination of this parameter - such as by the formula of A. P. Karnaukhov [2] with the use of a capillary model of a porous body having channels of variable cross section - is difficult due to the absence of literature data on the geometric form of pore channels.

Below we present a method of calculating $\delta_{a y}^{m}$ based on a capillary model with channels of variable cross section. The method employs the familiar [1] assumption of equality of porosity $P$ and the specific surface of the pores $S_{S p}$ in the model and the porous body (these assumptions in turn imply equality of the hydraulic diameters $\delta^{h}$ of the model and body, determined from the formula $\delta^{h}=4 \mathrm{P} / \mathrm{S}_{\mathrm{sp}}$ ). The value of $\delta^{\mathrm{m}}$ av is assumed to be equal to the size of the cross section of capillary tubes $\delta^{c}$. The choice of the capillary model is based on the relation $\delta_{a v}^{m}<\delta^{h}$, which is obvious for porous media. Since of all the models with capillary tubes of constant cross section, this condition (with allowance for the equality $\delta_{a v}^{m}=\delta^{c}$ ) is satisfied only by the model with intersecting tubes, we used a grid model of the cubic type [3, 4] to calculate $\delta_{\mathrm{av}}^{\mathrm{m}}$.

In performing the calculations, we used the effective length coefficient $\mathrm{K}_{\mathcal{Z}}$ to account for the effect of mutual intersection of the tubes on their total volume and lateral surface area. This coefficient is equal to the ratio of the length of the generatrix of the lateral surface of the tube to the length of its axial line. It has been established that for the chosen model, the dependences of $\mathrm{K}_{2}$ on the opening m and the porosity of the model are described by the equations:

$$
\begin{gather*}
K_{l}=1-V / \bar{m} ;  \tag{1}\\
2 K_{l}^{3}-3 K_{l}^{2}=\mathrm{P}-1, \tag{2}
\end{gather*}
$$

while the hydraulic diameter of the model is connected with the cross section of the capillary tubes by the formula

$$
\begin{equation*}
\delta^{\mathrm{h}}=\frac{1+2 K_{1}}{3 K_{l}} \delta^{\mathrm{c}} . \tag{3}
\end{equation*}
$$

We change over in (3) from the model parameters to the parameters of the porous material on the basis of the assumptions made and the function $\delta^{h}=f\left(P ; S_{s p}\right)$, in which $S_{s p}$ of the porous material is represented in the form [2]

$$
\begin{equation*}
S_{\mathrm{sp}}=\frac{K_{\mathrm{p} \gamma}}{d_{\mathrm{p}}}(1-\mathrm{p}) . \tag{4}
\end{equation*}
$$

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In the solution of Eq. (3) for $\delta^{c}$, with the replacement of the latter by $\delta_{a v}^{m}$ and with allowance for (4), we obtain the following:

$$
\begin{equation*}
\delta_{\mathrm{av}}^{\mathrm{m}}=\frac{12 K_{l}}{\left(1+2 K_{l}\right) K_{\mathrm{p}} \gamma} \frac{\mathrm{p}}{1-\mathrm{P}}-d_{\mathrm{p}} . \tag{5}
\end{equation*}
$$

Equation (5) was used to calculate $\delta_{a v}^{m}$ for recticular porous materials (RPM) [1]. Since the wire netting used to make these materials is obtained from wire of circular cross section, we took $K_{p}=4$ in the calculations and we took the diameter of the wire $d_{w i}$ as the characteristic dimension of the particles. In a number of cases, RPM's are made of netting obtained by interweaving two wires of different diameters (the warp and weft in filter gauze) or strands of wire of a single diameter. For these materials, we chose the diameter of an equivalent fiber as the characteristic dimension. This diameter is determined from the formula

$$
\begin{equation*}
d_{e}=4 V_{\Sigma} / S_{\Sigma} . \tag{6}
\end{equation*}
$$

In calculating the equivalent diameter of a strand of wires, the quantity $S_{\Sigma}$ was taken equal to the area of the outside surface of the strand.

It was established on the basis of (6) that for a hexagonal packing of wires in a strand, the interpolational dependence of the equivalent diameter of the strand on the number $N_{w i}$ of wires in the strand can be determined with an error no larger than $5 \%$ from the expression $d_{e}=0.6 \sqrt{N_{w i}+1}$. In the case of cloth filter gauze and a one-sided twill weave

$$
\begin{equation*}
d_{\mathrm{e}}=d_{\mathrm{w}}\left(A+K_{1} \bar{d}_{\mathrm{w}}^{2}\right)\left(A+K_{1} \bar{d}_{\mathrm{waz}}{ }^{-1},\right. \tag{7}
\end{equation*}
$$

where

$$
\begin{array}{r}
A=\left(K_{2}+\cos \varphi\right) n_{\mathrm{we}} n_{\mathrm{wa}}^{-1}+d_{\mathrm{we}}\left(1+\bar{d}_{\mathrm{wa}}\right) n_{\mathrm{we}}{ }^{\varphi l^{-1}} ; \\
\varphi=\arcsin \left[\left(d_{\mathrm{wa}}+d_{\mathrm{we}}\right) n_{\mathrm{wa}}^{-1} l^{-1}\right] ; \bar{d}_{\mathrm{wa}}=d_{\mathrm{wd}} d_{\mathrm{we}} .
\end{array}
$$

For gauze of plain weave, $K_{1}=0, K_{2}=1$; for gauze with a one-sided twill weave, $K_{1}=1, K_{2}=$ 0.

Strands of wires have a low stability. Thus, when a packet of gauzes is pressed, the gauzes spread out. This spreading is accompanied by an increase in the area of the outside surface of the gauzes and a reduction in the equivalent diameter. The dependence of $d_{e}$ of stzands with different numbers of wires on the degree of their compression (Fig. 1) can be found from Eq. (6).

The value of the coefficient $y$ depends on the character of the contacts between particles in the packing. At values of compression $\varepsilon$ of a multilayered packet of gauzes not exceeding a certain critical value $\varepsilon_{c r}$, the packet will be consolidated mainly as a result of bending of the wires, while the wires will contact each other along the generatrix of a cylinder or at a point. Given this type of contact, there will not be any reduction in the total surface area of the wires in the packet $\mathrm{S}_{\mathrm{wi} \mathrm{\Sigma}}$, so that $\gamma=1.0$. In the region $\varepsilon>\varepsilon_{\mathrm{cr}}$, the bending strain of the wires is transformed into strain associated with their spreading. Here, small areas of contact will be formed and, thus, there will be a reduction in $\mathrm{S}_{\mathrm{wiL}}$. In this case, $\gamma<1$. 0 .

The quantity $\varepsilon_{c r}$ is dependent on the geometric characteristics of the gauzes, the number of layers $\mathrm{N}_{\mathrm{a}}$, and the method used to arrange the gauzes in the packet. It was established that the mathematical expectation and the dispersion of $\varepsilon_{c r}$ are determined by the following respective formulas:
for packets of cloth filter gauzes of one-side twill weave arranged so that the filaments of the warp in adjacent layers are rotated by $90^{\circ}$ :

$$
\begin{gather*}
\left.m_{\left[\varepsilon_{\mathrm{cr}}\right]}\right]=\left(1-\frac{d_{\mathrm{wa}}}{l} n_{\mathrm{wa}}\right)\left(1-\frac{2}{N_{l \mathrm{a}}}\right)\left(1+2 \frac{d_{\mathrm{we}}}{d_{\mathrm{wa}}}\right)^{-1} ;  \tag{8}\\
D\left[\varepsilon_{\mathrm{cr}}\right]=\left(N_{l \mathrm{a}}-2\right) \frac{d_{\mathrm{wa}}}{l} n_{\mathrm{wa}}\left(1-\frac{d_{\mathrm{wa}}}{l} n_{\mathrm{wa}}\right)\left(1+2 \frac{d_{\mathrm{we}}}{d_{\mathrm{wa}}}\right)^{-2} N_{\mathrm{la}}^{-2} ; \tag{9}
\end{gather*}
$$

for packets of cloth gauzes with square cells arranged so that the filaments in adjacent layers are rotated $45^{\circ}$ :

$$
\begin{equation*}
m\left[\varepsilon_{\mathrm{cr}}\right]=(1-d / t)\left(1-1 / N_{l \mathrm{a}}\right) ; \tag{10}
\end{equation*}
$$



Fig. 1. Dependence of the ratio of the equivalent diameter of a strand to the diameter of the wires on the degree of compression of the strand and the number of wires in it. The numbers next to the curves denote the number of wires in the strand.


Fig. 2. Dependence of the dimensionless ratio $\delta^{\mathrm{m}} / \mathrm{d}$ on the porosity of the RPM: I) calculation with Eq. (5); II) calculation with Eq. (12); points) experimental data: a) RPM of cloth gauzes; 1) S 685 ; 2) S 450 ; 3) S 200 ; 4) S 120 ; 5) P 24 ; 6) P 60 ; 7) P80; 8) P90; b) RPM of woven gauzes with the following numbers of in a strand $N_{w i}$ : 1) 1 ; 2) 3 ; 3) 6 ; 4) 11 ; 5) 20 ; 6) 30 ; 7) 40.

$$
\begin{equation*}
D\left[\varepsilon_{\mathrm{cr}}\right]=\frac{N_{l \mathrm{a}}-1}{2 N_{l \mathrm{a}}^{2}} \frac{d}{t}(1-d / t) \tag{11}
\end{equation*}
$$

The stability of the properties of an RPM decreases with a reduction in the number of layers in the packet, so that it is customary in making these materials that $\mathrm{N}_{\mathrm{la}} \geq 4$. Also, in accordance with Eqs. (8) and (10), $m\left[\varepsilon_{c r}\right] \geq 0.14$ and $m\left[\varepsilon_{c r}\right] \geq 0.35$, respectively. Since an increase in the compression of the packet leads to a reduction in the porosity and permeability of the material, in making $R P M^{\prime} s$ the value of $\varepsilon$ usually does not exceed 0.25 for packets of twill-weave gauzes and 0.55 for packets of gauzes with square cells. Allowing for the foregoing when performing calculations with Eq. (5), we can take $\gamma=1.0$.

Experimental data obtained by expelling fluid from pores [1] for different types of RPM's is compared in Fig. 2 with results calculated from Eq. (5) and analyzed in the form of the dependence of the dimensionless ratio of $\delta_{a v}^{m}$ to the equivalent diameter of the filament on the porosity of the material (line I).

The value of $\delta^{h}$ was taken in several cases as the theoretical value of the mean pore size. Figure 2a (curve II) shows the results of calculation of $c^{h}$ of an RPM from the formula

$$
\begin{equation*}
\delta^{\mathrm{h}}=\frac{4}{K_{\mathrm{p}} \hat{V}} \frac{\mathrm{P}}{1-\mathrm{P}} d_{\mathrm{e}} ; \tag{12}
\end{equation*}
$$

this formula having been obtained on the basis of the relation $\delta^{h}=f\left(P ; S_{s p}\right)$ and Eq. (4).
Equation (12) is a special case of Eq. (5) with $K_{\eta}=1$ and coincides with the familiar relations proposed by Kozeni [1] and Karnaukhov [2] to calculate the mean size of pores in porous materials from a model with nonintersecting capillary tubes. For this model, ${ }^{h}$ is identical to $\delta_{a v}$.

## NOTATION

$d_{p}$, characteristic dimension of particles making up the porous material; ${ }_{\text {wa }}$, diameter of warp wire; $d_{\text {we }}$, diameter of weft wire; $K_{1}$ and $K_{2}$, coefficients; $K_{p}$, particle form factor; $n_{\text {wa }}$, number of wires of the warp over the length $l ;{ }^{n}$ we, number of wires of the weft over the length $Z$; $S_{\Sigma}$, total area of lateral surface of wires $\frac{\mathrm{We}^{\prime}}{}$ an arbitrary piece of gauze; $t$, spacing of wires in gauze; $V_{\Sigma}$, total volume of wires in an arbitrary pieze of gauze; $\gamma$, accessibility coefficient, accounting for the reduction in the total surface area of the particles in the material due to their mutual obstruction; $\varphi$, angle of contact of warp with weft.

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diffusion of a passive Impurity in a porous medium.
FRACTAL MODEL IN THE CASE OF AN UNSATURATED MEDIUM
A. B. Mosolov and O. Yu. Dinariev

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The theory of fractal sets is used to describe convective diffusion in a partly-saturated porous medium.

The study of the diffusion of a passive impurity being transported by a liquid or gas in a porous medium is one of the main approaches used to investigate flows in porous materials. By introducing a neutral indicator (such as radioactive isotopes) into the flow and following its distribution, it is possible to obtain a large quantity of information on the motion and mixing of the fluid. Besides isotopes, the neutral indicator may be a pigment or even temperature if the investigator is interested in processes involving convective heat transfer. Helium is of ten used as the indicator in the study of transport processes in gases.

It is particularly interesting to study convective diffusion in a partly saturated medium, since in this case it is possible to obtain information not only on the flow itself, but also on the geometric characteristics of the regions occupied by a single phase. It is understood that diffusion becomes "anomalous" in an unsaturated medium and differs appreciably from both normal molecular diffusion and convective diffusion in a completely saturated porous medium, since the diffusion coefficient depends not only on the dynamic characteristics of the

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[^0]:    All-Union Scientific-Research Institute of Natural Gases, Moscow. Translated from In-zhenerno-Fizicheskii Zhurnal, Vol. 53, No. 4, pp. 612-617, October, 1987. Original article submitted June 20, 1986.

